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# NOTE ON THE THEORY OF ELECTRIC ABSORPTION.

By H. A. Rowland.

In experimenting with Leyden jars, telegraph cables and condensers of other forms in which there is a solid dielectric, we observe that after complete discharge a portion of the charge reappears and forms what is known as the residual charge. This has generally been explained by supposing that a portion of the charge was conducted below the surface of the dielectric, and that this was afterwards conducted back again to its former position. But from the ordinary mathematical theory of the subject, no such consequence can be deduced, and we must conclude that this explanation is false. Maxwell, in his "Treatise on Electricity and Magnetism," vol. 2, chap. x, has shown that a substance composed of layers of different substances can have this property. But the theory of the whole subject does not yet seem to have been given.

Indeed, the general theory would involve us in very complicated mathematics, and our equations would have to apply to non-homogeneous, crystalline bodies in which Ohm's law was departed from and the specific inductive capacity was not constant; we should, moreover, have to take account of thermo-electric currents, electrolysis, and electro-magnetic induction. Hence in this paper I do not propose to do more than to slightly extend the subject beyond its present state and to give the general method of still further extending it.

Let us at first, then, take the case of an isotropic body in general, in which thermo-electric currents and electrolysis do not exist, and on and in which the changes of currents are so slow that we can omit electro-magnetic induction. The equations then become *

$$\frac{d}{dx}\left(\kappa\frac{dV}{dx}\right)+\frac{d}{dy}\left(\kappa\frac{dV}{dy}\right)+\frac{d}{dz}\left(\kappa\frac{dV}{dz}\right)+4\pi\rho=0$$

$$\frac{d}{dx}\left(k\frac{dV}{dx}\right)+\frac{d}{dy}\left(k\frac{dV}{dy}\right)+\frac{d}{dz}\left(k\frac{dV}{dz}\right)-\frac{d\rho}{dt}=0$$

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* Maxwell's Treatise, Art. 825.

in which $\varkappa$ is the specific inductive capacity of the substance, $k$ the electric conductivity, $V$ the potential, $\rho$ the volume density of the electricity, and $t$ the time.

The subtraction of one equation from the other gives

$$(1)\qquad \frac{dV}{dx}\frac{d}{dx}\left(\log\frac{k}{\varkappa}\right)+\frac{dV}{dy}\frac{d}{dy}\left(\log\frac{k}{\varkappa}\right)+\frac{dV}{dz}\frac{d}{dz}\left(\log\frac{k}{\varkappa}\right)-\frac{1}{k}\frac{d\rho}{dt}-\frac{4\pi\rho}{\varkappa}=0.$$

To introduce the condition that there shall be no electric absorption, we must observe that when that phenomenon exists, a charge of electricity appears at a point where there was no charge before; in other words, the relative distribution has been changed. Hence, if the relative distribution remains the same, no electric absorption can take place. Our condition is, then,

$$\frac{\rho}{\rho'}=c,$$

where $c$ is independent of $t$, and $\rho$ and $\rho'$ are the densities at the points $x$, $y$, $z$, and $x'$, $y'$, $z'$. This gives

$$\frac{d}{dt}\left(\log\frac{\rho}{\rho'}\right)=0,$$

$$(2)\qquad\text{or}\qquad \frac{1}{\rho}\frac{d\rho}{dt}=\frac{d}{dt}\left(\log\frac{\rho}{\rho_0}\right)=-c,$$

where $c$ is a function of $t$ only and not of $x$, $y$, $z$, and $\rho_0$ is the value of $\rho$ at the time $t=0$. As we have

$$\frac{1}{m}\frac{dV}{dn}\frac{dm}{dn}=\frac{dV}{dx}\frac{d}{dx}\left(\log\frac{k}{\varkappa}\right)+\frac{dV}{dy}\frac{d}{dy}\left(\log\frac{k}{\varkappa}\right)+\frac{dV}{dz}\frac{d}{dz}\left(\log\frac{k}{\varkappa}\right),$$

where $m=\frac{k}{\varkappa}$ and $n$ is a line in the direction of the current at the given point, equation (1) becomes

$$\frac{1}{m}\frac{dV}{dn}\frac{dm}{dn}-\frac{1}{k}\frac{d\rho}{dt}-\frac{4\pi\rho}{\varkappa}=0,$$

From equation (2) $$\rho=\rho_0\,\varepsilon^{-\int_0^t c\,dt},$$

and hence

$$\frac{1}{m}\frac{dV}{dn}\frac{dm}{dn}+\rho_0\varepsilon^{\int_t^0 c\,dt}\left(\frac{c}{k}-\frac{4\pi}{\varkappa}\right)=0.$$

If we denote the strength of current at the point by $S$, we have

$$S = -k\frac{dV}{dn},$$

and

$$(3) \qquad \frac{1}{cm - 4\pi m^2}\,\frac{dm}{dn} = +\frac{\rho_0}{S}\,\varepsilon^{\int_t^0 cdt} ;$$

this equation (3) gives the value of $\frac{k}{\kappa} = m$ at all points of the body and at all times so that the phenomenon of electric absorption shall not take place As this equation makes $m$ a function of $x$, $y$, $z$, $S$ and $t$, the relation in general is entirely too complicated to ever apply to physical phenomena, without some limitation. Firstly then, as $c$ is only an arbitrary function of $t$, we shall assume that it is constant;

$$\therefore \quad \frac{1}{cm - 4\pi m^2}\,\frac{dm}{dn} = +\frac{\rho_0 \varepsilon^{-ct}}{S} .$$

The most important case is where $m$ is a constant. Then

$$\frac{dm}{dn} = 0,$$

and

$$c = 4\pi m, \qquad S = S_0\,\varepsilon^{-ct}, \qquad \rho = \rho_0\,\varepsilon^{-ct} .$$

In this case, therefore, we see that both the electrification and the currents die away at the rate $c$. The case where Ohm's law is true and the specific inductive capacity is constant is included in this case, seeing that when $k$ and $\kappa$ are both constants their ratio, $m$, is constant. But it also includes the cases where $k$ and $\kappa$ are both the same functions of $V$, $S$, or $x$, $y$, $z$, seeing that their ratio, $m$, would be constant in this case also.

When $m$ is not constant, the chances are very small against its satisfying equation (4).

*Hence, we may in general conclude, that electric absorption will almost certainly take place unless the ratio of conductivity to the specific inductive capacity is constant throughout the body.*

This ratio, $m$, may become a variable in several manners, as follows:

*1st manner.*—The body may not be homogeneous. This includes the case, which Maxwell has given, where the dielectric was composed of layers of different substances.

*2d manner.*—The body may not obey Ohm's law; in this case $k$ would be variable.

*3d manner.*—The specific inductive capacity, $\varkappa$, may vary with the electric force.

It is to be noted that the cases of electric absorption which we observe are mostly those of condensers formed of two planes, or of one cylinder inside another, as in a telegraph cable. Our theory shows that different explanations can be given of these two cases.

The case of parallel plates does not admit of being explained, except on the supposition that $m$ varies in the first manner above given, or in this manner in combination with the others, for we can only conceive of the conductivity and the specific inductive capacity as being functions of the ordinate or of the electric force. As the latter is constant for all points between the plates, $m$ would still be constant although it were a function of the electric force, and thus electric absorption would not take place.

We may then conclude that in the case of parallel plates, omitting explanations based on electrolysis or thermo-electric currents, the only explanation that we can give at present is that which depends on the non-homogeneity of the body, and is the case which Maxwell has given in the form of two different materials. Our equations show that the form of layers is not necessary, but that any departure from homogeneity is sufficient. It is to be noted that the homogeneity, which we speak of, is electrical homogeneity, and that a mass of crystals with their axes in different directions would evidently not be electrically homogeneous and would thus possess the property in question. In the case of glass it is very possible that this may be the case and it would certainly be so for ice or any other crystalline substance which had been melted and cooled.

In the case of hard india rubber, the black color is due to the particles of carbon, and as other materials are incorporated into it during the process of manufacture, it is certainly not electrically homogeneous.

As to the ordinary explanation that the electricity penetrates a little below the surface and then reappears again to form the residual charge, we see that it is in general entirely false. We could, indeed, form a condenser in which the surface of the dielectric would be a better conductor than the interior and which would act thus. But in general, the theory shows that the action takes place throughout the mass of the dielectric, where that is of a fine grained structure and apparently homogeneous, as in the case of glass, and consists of a polarization of every part of the dielectric.

To consider more fully the case of a condenser made of parallel plates, let us resume our original equations. Without much loss of generality we can assume a laminated structure of the substance in the direction of the plane $YZ$, so that $m$ and $V$ will be only functions of the ordinate $x$. Our equations then become

$$\frac{d}{dx}\left(\varkappa\frac{dV}{dx}\right)+4\pi\rho=0,$$

$$\frac{d}{dx}\left(k\frac{dV}{dx}\right)-\frac{d\rho}{dt}=0.$$

Eliminating $\rho$ we find

$$\frac{1}{4\pi}\frac{d}{dt}\frac{d}{dx}\left(\varkappa\frac{dV}{dx}\right)+\frac{d}{dx}\left(k\frac{dV}{dx}\right)=0.$$

Now let us make $p=\varkappa\dfrac{dV}{dx}$, and as $t$ and $x$ are independent, we find on integration,

$$\frac{d}{dt}(p-p_0)+4\pi\,(pm-p_0m_0)=0,$$

where $p_0$ is the value of $p$ for some initial value of $x$, say at the surface of the condenser, and is an arbitrary function of $t$, seeing that we may vary the charge at the surface of the body in any arbitrary manner. This equation establishes $p$ as a function of $m$ and $t$ only, and as we have

$$\rho=-\frac{1}{4\pi}\frac{dp}{dx},$$

$\rho$ will also be a function of these only.

Let us now suppose that at the time $t=0$, the condenser is charged, having had no charge before, and let us also suppose that the different strata of the dielectric are infinitely thin and are placed in the same order and are of the same thickness at every part of the substance, so that a finite portion of the substance will have the same properties at every part.

In this case $m$ will be a periodic function of $x$, returning to the same value again and again. As $\rho$ is a function of this and of $t$ only, at a given time $t$, it must return again and again to the same value as we pass through the substance, indicating a uniform polarized structure throughout the body.

This conclusion would have been the same had we not assumed a laminated structure of the dielectric. In all other cases, except that of two planes, electric absorption can take place, as we have before remarked, even in perfectly homogeneous bodies, provided that Ohm's law is departed from or that the electric induction is not proportional to the electric force, as well as in non-homogeneous bodies. But where the body is thus homogeneous, electric absorption is not due to a uniform polarization, but to distinct regions of positive and negative electrification.

In the whole of the investigation thus far we have sought for the means of explaining the phenomenon solely by means of the known laws of electric induction and conduction. But many of the phenomena of electric absorption indicate electrolytic action, and it is possible that in many cases this is the cause of the phenomenon. The only object of this note is to partially generalize Maxwell's explanation, leaving the electrolytic and other theories for the future.